

March 29, 1832.

GEORGE RENNIE, Esq. Vice President, in the Chair.

The following Report, drawn up by the Rev. William Whewell, M.A. F.R.S., and John William Lubbock, Esq. M.A. V.P. and Treasurer R.S., on Professor Airy's Paper, read before the Royal Society on November 24, 1831, and entitled, "On an Inequality of Long Period in the Motions of the Earth and Venus," was read.

Report.

The object of this memoir is similar to that of Laplace's celebrated investigation of the great inequality of Jupiter and Saturn, announced in the *Memoirs of the Academy of Sciences* for 1784, and given in the volume for the succeeding year. The occasion of that investigation was an acceleration of the mean motion of Jupiter and a retardation of that of Saturn,—which inequalities in the motions of the two planets Halley had discovered by a comparison of ancient and modern observations : and Laplace showed, in the *Memoirs* just referred to, that inequalities like those thus noticed would arise from the action of gravitation ; that they would reach a considerable amount in consequence of twice the mean motion of Jupiter being very nearly equal to five times the mean motion of Saturn ; and that their period would be nearly 900 years. The occasion of the investigation of Professor Airy was an inequality in the sun's actual motion, as compared with Delambre's *Solar Tables*, which appeared to result from a comparison of late observations with those of the last century,—as Professor Airy has explained in a memoir published in the *Philosophical Transactions* for 1828. This comparison having convinced him of the necessity of seeking for some inequality of long period in the earth's motion, it was soon perceived that such an inequality would arise from the circumstance that 8 times the mean motion of Venus is very nearly equal to 13 times the mean motion of the earth. The difference is 1,675 centesimal degrees in a year, —from which it follows, that if any such inequality exist, its period will be about 240 years.

To determine whether such an inequality arising from the action of gravitation, amounts to an appreciable magnitude, is a problem of great complexity and great labour. The coefficient of the term will be of the order 13 *minus* 8, or 5, when expressed in terms of the excentricities of the orbits of the Earth and Venus, and their mutual inclination ; all which quantities are small ; and the result would therefore, on this account, be very minute. But in the integrations by which the inequality is found, the small fraction expressing the difference of the mean motions of the planets enters twice as a divisor ; and by the augmentation arising from this and other parts of the process, the term receives a multiplier of about 2,200,000. In the corresponding step of the investigation of the great inequality of

Jupiter and Saturn, it was only necessary to take terms of the 3rd order of smallness, and the multiplier by which the terms are augmented has 30^2 instead of 240^2 for its factor.

In the present state of physical astronomy methods exist by which the results of the law of universal gravitation in the planetary system may be obtained to any degree of accuracy, by calculating in succession the terms of successive orders of minuteness, the order being estimated according to the powers and products of the excentricities of the orbits. But it is well known, that in the actual application of these methods, the number of the terms arising from the combination of the several series which occur, and the complexity of the operations by which the coefficients of these terms are to be deduced, increase so rapidly in passing beyond the lower orders of inequalities, that the calculation is difficult and laborious.

The numerical calculation of a perturbation depending on the 5th powers of the excentricities has not been executed, so far as we are aware, except in the case of the great inequality of Jupiter and Saturn, where, as Laplace states (*Mécanique Céleste*, p. ii. liv. 6. 9°), this labour, “*pénible par son excessive longueur*,” has been performed by Burckhardt with a scrupulous attention. And no calculation of a new inequality of a high order, requiring to be placed in the planetary tables, with a new argument, has been published since that of the great inequality by Laplace in 1784.

One of the main parts of the labour of such calculations consists in obtaining the successive terms of a certain quantity on which the perturbing forces depend, and which in the *Mécanique Céleste* is called *R*. This quantity is a function of the positions of the two planets which affect each other, and involves the reciprocal of the distance of the two bodies. It is to be expanded according to the powers and products of the excentricities and inclination of the two orbits, its successive terms having as factors the cosines of certain angles, all of which increase proportionally to the time.

It may be expanded by Taylor's theorem applied to several variables, according to powers of the excentricities, and f^2, f being the sine of half the inclination*.

In this expansion the coefficients of the cosines of the different arguments are functions of certain quantities *A* or *b* (according to the notation of the *Mécanique Céleste*), and of the partial differential coefficients of these quantities with regard to *a* and *a'*, the radii of the orbits; admitting however of reduction so as to contain the differential coefficients with regard to one of these quantities only.

The quantity *b* is a function of $\frac{a'}{a}$, and has several values distinguished by different indices: these are connected by certain well-known equations of condition. The author of the present memoir obtains the development of *R* in terms of quantities *C*, sym-

* Laplace uses *s*, the tangent of the inclination. Burckhardt expresses himself to be in great doubt what function of the angle it is best to take.

metrical functions of a and a' ; and the different quantities C are of course connected by equations of condition similar to the others.

The general development of R , given in the third volume of the *Mécanique Céleste*, extends only to the terms depending on the squares of the excentricities. Burckhardt carried the development much further in the *Mémoires de l'Institut* for 1808; but Professor Airy's formulæ are not immediately comparable with his, on account of the employment of C instead of b . The formulæ of Burckhardt include however only 6 out of 12 of Professor Airy's coefficients, in consequence of the omission of terms depending on the inclination, by the former mathematician.

In the expansion of R , the terms which proceed according to the powers of the excentricities and of the sine of half the inclination (e, e', f), involve cosines of the multiples of V, T , and $V-T$, (V and T indicating the mean motions of Venus and the Earth with the addition of certain constant quantities.) By expanding also the variations of the radii vectores, which occur in these terms, according to powers of e, e' , the cosines of multiples of the arcs V and T again enter these terms. Hence the series will finally contain the products of the cosines of the three kinds of arcs, namely, multiples of $V-T, V$ and T ; which, as is well known, may be expressed in terms of the cosines of their sums and differences. These last arcs will produce various combinations of the form $pV - qT$. Of these we are to select those in which the arc is $13V - 8T$; for such terms will, in the calculation of the perturbations, be divided by very small quantities (namely, either $\frac{1}{240}$ or $\frac{1}{240^2}$), and may thus produce results of appreciable magnitude.

It may serve to assist us in forming a judgement concerning the place of Professor Airy's memoir among the laborious calculations of physical astronomy, if we compare it with the investigation of the great inequality of Jupiter and Saturn, as originally given by Laplace (*Mém. Acad.* 1785), the undertaking to which it has the closest analogy among such researches*.

The number of terms arising from the combination of the three arcs above mentioned, $V-T, V$, and T , which give $13T - 8V$, is considerable; but many of them are rejected on account of their coefficients going beyond the 5th order. The following *six* are retained and have their value calculated: those in which $V-T, V, T$ are respectively 8, 5, 0; 9, 4, 1; 10, 3, 2; 11, 2, 3; 12, 1, 4; 13, 0, 5. In the Jovi-Saturnian inequality, if V and T still refer to the mean motions of the exterior and interior planets, *four* combinations are taken, namely, those in which $V-T, V, T$ are respectively 2, 3, 0; 3, 2, 1; 4, 1, 2; 5, 0, 3.

The number of terms of the calculation depends also upon the

* The comparison is here made with the investigation of the *principal term* only of the Jovi-Saturnian inequality, as the most celebrated and most analogous, not as the most laborious or most recent, of similar investigations.

order to which it is carried. In Professor Airy's investigation there are, to each angular factor, *twelve* terms arising from the combinations of e, e', f ; in that of the inequality of Jupiter to the 3rd order, there are *four* such terms.

The number of the coefficients C , of the series for R , which it is requisite to calculate, depends both upon the number of effective combinations of arcs and the number of effective combinations of e, e', f . Hence the number of such coefficients, and their differentials, which Professor Airy's calculation demands, is very considerable. In the calculation for Jupiter, Laplace uses 6 coefficients b , and 14 of their differentials, in which however 28 differentials are virtually obtained. Professor Airy has occasion to use 70 values of C , the corresponding quantity in his process, and 98 of its differential coefficients; these quantities being calculated to a number of places from 7 downwards.

The calculation of the inequalities of the motions of Venus and the Earth, from the numbers thus obtained, requires the combination of these numbers with others depending on the excentricities, inclination, perihelia, and nodes of the orbits; and contains, as has already been said, 12 compound terms in the present investigation, and 4 in that of Laplace.

The greatest amount of the inequality thus explained by Laplace, was $20''$ for Jupiter and $47''$ for Saturn. The effect of the inequality examined in the memoir before us, would give an error in the geocentric longitude of Venus of between $20''$ and $30''$, if the mean motions of the Earth and Venus were determined, by comparing the observations about Bradley's time with the observations of a few years ago; and if the result were applied to calculate the next transit of Venus (in 1874).

The method adopted by Professor Airy in this investigation offers some peculiarities. There are two principal methods which may be employed in such problems: one is the method of *direct solution*, according to which the equations on which the inequalities depend are solved directly, and the values obtained by the first approximate solutions are substituted in the terms before neglected, in order to obtain a new solution. The other method is that of the *variation of parameters* (developed by Lagrange), according to which the planet, at any moment of time, is conceived to be moving in an ellipse, and the alterations are investigated which the elements of this ellipse must continually undergo, in order that the real motion may result. The former of these methods is the one which has generally been employed in calculating all inequalities of the planets except secular ones, and is used by Laplace in the theory of Jupiter and Saturn. In the present memoir the author has adopted the method of the variation of parameters, and he states his opinion that this, or some similar method, will ere long be adopted in the planetary theories, to the exclusion of other methods.

In one instance the author has introduced an alteration into the formulæ given for the variation of the elements by those who have

hitherto employed the theory of Lagrange. The differential $\frac{dR}{de}$, in Lagrange's method, implicitly includes the differential of R with regard to the mean motion. But it has been shown by Lagrange himself, and since him by others, that we may finally omit the term depending on the variation of n , if we use $\int n dt$ throughout instead of nt . This reduction Professor Airy does not adopt, retaining explicitly the term $\frac{dR}{dn}$.

This difference in the formulæ is equivalent to a change in the meaning of the term *epoch*. In determining the longitude of the epoch of the instantaneous ellipse, Laplace, and others who have followed Lagrange, fix it by supposing this longitude to be the angle which we must add to the angle described by the *variable* elliptical mean motion since the origin of the time, in order to have the mean anomaly in the ellipse. Professor Airy assumes the longitude of the epoch to be that angle, which we must add to the angle described by the instantaneous elliptical mean motion, considered *constant* since the origin of the time, in order to have the mean anomaly: the mean motion from the perihelion is, on the first supposition, $\int n dt + \varepsilon$, on the second, $nt + \varepsilon$. In the results of the calculations in these two ways there is no discrepancy, the difference of the formulæ and the difference of the suppositions necessarily balancing each other.

It may be observed, that according to the method of the variation of parameters, a large portion of the inequality, in the present instance, falls upon the longitude of the epoch. The coefficient of this inequality is something more than $2''$; which produces nearly the same maximum amount in the longitude of the planet, the effects of the variations of the other elements being insensible.

In investigations of such extent and complexity as the one now before us, the selection of notation is a matter of considerable importance, in order to obtain the greatest possible degree of clearness and brevity. In all cases when nothing is gained by the change, it is convenient to the reader that the notation should conform to the best established works already published. Professor Airy has in general used the notation of the *Mécanique Céleste*. He has, however, introduced a new notation, in order to express in an abbreviated manner the differentials of the quantities C , taken m times with regard to one major semiaxis, and n times with regard to the other, and multiplied respectively by the m th and n th powers of these semiaxes; these products occurring so frequently, that the adoption of a short symbol for them, $(m, n) C$, saves a great quantity of very repulsive labour.

Another abbreviation employed by Professor Airy respects the angles. In the development of the terms arising from the successive steps of the expansion of R , we obtain terms such as $e^2 \cos 2V$ and $e^3 \cos 3T$, multiplied by others, such as $\cos(11V - 11T)$; and by the resolution of such products we obtain the cosines of the sums and differences of these angles. But it appears that the sums

alone are useful. Hence it follows, that if we have, at a certain step of the process, $\cos (2V + 3T)$, its coefficient must be $e^2 e'^3$, and this may be written $e^2 e'^3 \cos (2 + 3)$ without fear of mistake; and this, when combined with such a term as $\cos (11V - 11T)$, will produce $e^2 e'^3 \cos (13 - 8)$. This mode of writing and operating is also a great saving of labour; for V and T consist of the mean motions, with several constant terms added or subtracted.

The author states that he has paid great attention to ensuring the accuracy of the work; having gone through the calculation by two different methods, and compared the values thus obtained, both in several intermediate steps, and in the final results.

We regard this paper as the first specific improvement in the solar tables made by an Englishman since the time of Halley, as valuable from the care which the author has employed in the numerical calculations, as well as for the sagacity he has displayed in the detection of an inequality so small, and of so large period; and we recommend its insertion in the Philosophical Transactions.

(Signed)

W. WHEWELL.

J. W. LUBBOCK.

April 5, 1832.

DAVIES GILBERT, Esq., M.A. Vice-President, in the Chair.

Marshall Hall, M.D., Archibald John Stephens, Esq., Sir William Russell, Bart., M.D., Sir David Barry, Knt., M.D., and Charles Boileau Elliott, Esq., were elected Fellows of this Society.

The following Report, drawn up by Samuel Hunter Christie, Esq., M.A. F.R.S., and John Bostock, M.D. V.P.R.S., on Mr. Faraday's paper, read before the Royal Society on December 15, 1831, and entitled "Experimental Researches in Electricity," was read,

Report.

In the first section of this paper, the author considers the induction of electricity in motion.

Shortly after the discovery by Oersted of the influence of electricity in motion on a magnetic needle, it was almost simultaneously discovered by Arago, Davy, and Seebeck, that iron became magnetic by induction from the connecting wire of a voltaic battery, or the passage of an electric current; but though the effects at first observed were afterwards greatly increased by peculiar arrangements, induction was in all cases restricted to iron. Arago's beautiful experiments on magnetic needles vibrating within metallic rings, and on the mutual action of all metals and magnets, when either is in motion, are undoubtedly instances of a peculiar magnetic induction in other metals than iron; but the very doubtful experiment of Ampère can scarcely be adduced as one. The singular results obtained by MM. Marianini, De la Rive, and Von Beek, referred to by our author, are probably due to electric induction. But none of these